B.Sc. II Mathematics
(SEMESTER

- III )
(MATHEMATICS)
Implemented from June - 2011
Paper - V (DIFFERENTIAL CALCULUS)

Unit - 1 : LIMITS AND CONTINUITY OF REAL VALUED

## FUNCTIONS

13 lectures
$1.1 \varepsilon-\delta$ definition of the limit of a function of one variable.
1.2 Basic properties of limits.
1.3 Continuous functions and their properties.
1.3.1 If $f$ and $g$ are two real valued functions of $a$ real variables which are continuous at $\mathrm{x}=\mathrm{c}$ then (a) $f+g$, (b) $f-g$, (c) f.g are continuous at $x=c$ and

$$
\begin{aligned}
& (d)_{f} \text { is } \\
& g_{\text {continuou }} x
\end{aligned}
$$

1.3.2 Composite function of two continuous functions is continuous.
1.3.3 If a function $f$ is continuous in a closed interval $[a, b]$ then it is bounded in $[a, b]$.
1.3.4 If a function $f$ is continuous in a closed interval $[a, b]$ then it attains its bounds at least once in $[a, b]$.
1.3.5 If a function $f$ is continuous in a closed interval $[a, b]$ and
$f(a), f(b)$ are of opposite signs then there exists $c \in[a, b]$ such that $\mathbf{f}(\mathrm{c})=\mathbf{0}$.
1.3.6 If a function $f$ is continuous in a closed interval $[a, b]$ and if $f(a) \neq f(b)$ then $f$ assumes every value between $f(a)$ and $f(b)$.
1.4 Classification of discontinuities ( First and second kind).
1.5 Uniform continuity.
1.5.1 A Real valued continuous function on $[a, b]$ is uniformly continuous on [a, b].
1.6 Sequential continuity.
1.6.1 A function $f$ defined on an interval $I$ is continuous at a point
$\mathrm{c} \in I$ if and only if for every sequence $\left\{C_{n}\right\}$ converging to $c$,
$\lim _{n \rightarrow \infty} f\left(C_{n}\right)=c$.
1.7 Differentiability at a point, Left hand derivative, Right hand derivative, Differentiability in the interval [a,b].
1.7.1 Theorem: Continuity is a necessary but not a sufficient condition for the existence of a derivative.

Unit - 2 : JACOBIAN
2.1 Definition of Jacobian and examples.
2.2 Properties of Jacobians.
2.2.1If $J$ is Jacobian of $u$, $v$ with respect to $x, y$ and $J$ is

Jacobian of $\mathrm{x}, \mathrm{y}$ with respect to $\mathrm{u}, \mathrm{v}$ then $\mathrm{JJ}^{\prime}=1$.
2.2.2 If $J$ is Jacobian of $u, v, w$ with respect to $x, y, z$ and $J^{\prime}$ is

Jacobian of $\mathrm{x}, \mathrm{y}, \mathrm{z}$ with respect to $\mathrm{u}, \mathrm{v}, \mathrm{w}$ then $\mathrm{JJ}^{\prime}=1$.
2.2.3 If $p, q$ are functions of $u, v$ and $u, v$ are functions of $x, y$ then prove that $\frac{\partial(p, q)}{\partial(u, v)}=\frac{\partial(p, q)}{\partial(x, y)} \cdot \frac{\partial(x, y)}{\partial(u, v)}$.
2.2.4 If $\mathbf{p}, \mathbf{q}, \mathbf{r}$ are functions of $\mathbf{u}, \mathbf{v}, \mathbf{w}$ and $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are functions of $\mathbf{x}, \mathbf{y}, \mathbf{z}$ then prove that $\frac{\partial(p, q, r)}{\partial(u, v, w)}=\frac{\partial(p, q, r)}{\partial(x, y, z)} \cdot \frac{\partial(x, y, z)}{\partial(u, v, w)}$.
2.2.5 Examples on these properties.

Unit - 3 : EXTREME VALUES

## lectures

3.1 Definition of Maximum, Minimum and stationary values of function of two variables.
3.2 Conditions for maxima and minima (Statement only) and examples.
3.3 Lagrange's method of undetermined multipliers of three variables.
3.3.1The extreme values of the function $f(x, y, z)$ subject to the condition $\phi(x, y, z)=0$.
3.3.2The extreme values of the function $f(x, y, z)$ subject to the conditions $\phi(x, y, z)=0$ and $\psi(x, y, z)=0$.
3.3.3Examples based on Lagrange' $s$ method of undetermined multipliers of three variables.
3.3.4 Errors and approximations.

## Unit-4: VECTOR CALCULUS

## 11 lectures

4.1 Differentiation of vector.
4.2 Tangent line to curve.
4.3 Velocity and acceleration.
4.4 Gradient, Divergence and Curl of a vector field.
4.5 Solenoidal vecor, Irrotational vector.
4.6 Conservative vector fields.

## REFERENCE BOOKS

1. B.S.Phadatare, U.H.Naik, P.V.Koparde, P.D.Sutar, P.D.Suryvanshi, M.C.Manglurkar, A Text Book Of Advanced Calculus Published by Shivaji University Mathematics Society (SUMS), 2005.
2. S.B.Kalyanshetti, S.D.Thikane, S.R.Patil, N. I. Dhanashetti, A Text Book Of Mathematics -Advanced Calculus Published by Sheth Publishers Pvt. Ltd. Mumbai.
3. T. M. Apostol, Mathematical Analysis, Narosa Publishing House, New Delhi, 1985.
4. R. R. Goldberg, Real Analysis, Oxford \& I. B. H. Publishing Co., New Delhi, 1970.
5. P. K. Jain and S. K. Kaushik, An Introduction to Real Analysis, S. Chand \& Co., New Delhi. 2000.
6. Gorakh Prasad, Differential Calculus, Pothishala Pvt. Ltd., Allahabad.
7. Murray R. Spiegel, Theory and Problems of Advanced Calculus, Schaum Publishing Co., New York.
8. N. Piskunov, Differential and integral Calculus, Peace Publishers, Moscow.
9. Shanti Narayan, A Course of Mathematical Anlaysis, S. Chand and Company, New Delhi.
10. P. N. and J. N. Wartikar, Applied Engineering Mathematics.
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Paper - VI (DIFFERENTIAL EQUATIONS)

## Unit - 1 : HOMONOGENEOUS LINEAR DIFFERENTIAL

EOUATIONS
8 lectures
1.1 General form of Homogeneous Linear Equations of Higher order and it's solution.
1.2 Equations reducible to homogeneous linear form.

Unit - 2 : SECOND ORDER LINEAR DIFFERENTIAL

## EOUATIONS

2.1 General form $: \frac{d^{2} y}{d x^{2}}+P \frac{d y}{d x}+Q y=R$.

## 17 lectures

2.2 Methods of solution:
2.2.1 Complete solution of Linear differential equation when one integral is known.
2.2.2 Transformation of the equation by changing the dependent variable ( Removable of $\mathbf{1}^{\text {st }}$ order derivative).
2.2.3 Transformation of the equation by changing the independent variable.
2.3 Method of variation of parameters.

## Unit - 3 : ORDINARY SIMULTANEOUS DIFFERENTIAL

## EOUATIONS

8 lectures
3.2 Simultaneous linear differential equations of the form

$$
\frac{d x}{P}=\frac{d y}{Q}=\frac{d z}{R} .
$$

3.3 Methods of solving simultaneous differential equations.
3.4 Geometrical Interpretation.

Unit - 4 : TOTAL DIFFERENTIAL EOUATIONS
lectures
4.1 Total differential equations [ Pfaffian differential equation ]
$\mathbf{P d x}+\mathrm{Qdy}+\mathrm{Rdz}=\mathbf{0}$.
4.2 Necessary condition for integrability of total differential equations.
4.3 The condition of exactness.
4.4 Methods of solving total differential equations :
(a) Method of Inspection,
(b) One variable regarding as a constant.
4.5 Geometrical Interpretation.
4.6 Geometrical Relation between Total differential equations and Simultaneous differential equations.

1. T.A.Teli, S.P.Thorat, A.D.Lokhande, S.M.Pawar, D.S.Khairmode, A Text Book Of Differential Equations Published by Shivaji University Mathematics Society (SUMS), 2005.
2. S.B.Kalyanshetti, S.D.Thikane, S.R.Patil, N. I. Dhanashetti, A Text Book Of Mathematics - Differential Equations Published by Sheth Publishers Pvt. Ltd. Mumbai.
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